the antenna-transmission line analog

a key to designing and understanding antennas

A practical, non-mathematical discussion of a technique used by professional engineers to design and analyze antenna performance — it is equally applicable to amateur antennas

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At the present state of the electronics art, the antenna represents the most rewarding, fun-filled, and low-cost area remaining for amateur experimentation. In addition, there is always the challenging possibility of making a real contribution to technology. In all cases, experimentation with radiators will invariably result in improved on-the-air signals.

Many creative amateurs who begin to investigate antennas, however, become frustrated. They have no difficulty understanding certain principles given in elementary treatments of antenna theory, and such initial knowledge carries them through an early fun period of cut and try; but some of the results obtained from such experiments are confusing and demand explanations not found in non-professional books on antennas. If the amateur persists in his experimentation and becomes seriously interested, he finally gets to a point where he wants to know – *before* stringing up more wire or guying up more sections of metal tubing – answers to questions such as, "How do I tailor my antenna design so I can tune over the entire band without the vswr on my feedline climbing to magnitudes into which my rig refuses to load? How much coil reactance does it take to resonate my old 75-meter vertical antenna on the 160-meter band? How efficient is my 20-meter center-loaded whip on the station wagon?" Beyond this, many hams would like to try out their own ideas for antennas but want to know beforehand, with reasonable accuracy, how their brainchild is going to perform on the air.

Giving up on the elementary texts, some of these same amateurs turn to the professional antenna literature for help but are usually stopped cold. Unless they are already engineers by training, they are taken aback by pages covered with the esoteric symbols of the higher mathematics: Fourier series and transforms, Bessel functions, Legendre polynomials, and everywhere copious use of the integral and differential calculus. Few

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amateurs want to go back to college in order to pursue a hobby. Even in the communications industry, many engineers feel there is something mysterious and scary about antennas and leave their design to a handful of specialists.

There is, however, a relatively easy way to avoid the need for using high powered mathematics while getting good, workable answers of engineering accuracy to your antenna questions; answers which will permit you to design complex antennas and predict their performance *before* you build them. The key to all this goes by the rather complicated sounding name of *Antenna/Transmission Line Analogue*, but the only complicated part about it is its name.

What the Transmission Line Analogue does is to permit an on-paper conversion of your own particular antenna or concept into an equivalent rf transmission line. Once done correctly, this analogue key cranks out answers like your own private computer terminal. You might suspect that because the analogue key dispenses with higher mathematics, it must be some inaccurate, slip-shod method shunned by the real professionals. This is far from true – the analogue key method is used daily by working professional antenna engineers to design commercial and military radiators of all types for use in the frequency spectrum from 10 kHz on up. In its most fundamental form it was used by the brilliant antenna theoretician Dr. Schelkunoff[1] in evolving his powerful mode theory of antennas. As you gain familiarity with the analogue key by usage, you will become positively ingenious in figuring out ways to extend its application into the most involved antenna situations.

To really use the analogue key effectively, however, you must first understand how and why it works and where it comes from. The material which follows may lead you back over some familiar territory, but the route is necessary to establish a certain basic way of thinking about antennas.

primary mode waves on cylindrical antennas

Fig. 1A shows a perfectly straight, center-fed cylindrical conductor magically suspended without support in free space. You may recognize it as the familiar doublet antenna, having a total length of $2h$ and a half length, h. Its uniform conductor diameter, d, is twice its radius, a. For the moment, forget just how long $2h$ or h is supposed to be in electrical degrees at the operating frequency f (hertz). This doublet antenna has two center input terminals labeled A and B. If you could connect a very accurate rf impedance bridge (complete with its own built-in signal source) directly to the antenna input terminals A - B, without disturbing the invisible fields surrounding it in space, the bridge would read out the doublet's input impedance $Z_{in(A,B)}$ in the form of two separate parts, R_{total} and jX. The R_{total} part is the real or resistive part of the entire complex impedance $Z_{in(A,B)}$; the X part with the lower-case j complex operator in front of it (as a label to make sure you separate it from the real part) is reactance. The bridge cannot tell you that the resistive part R_{total} is really made up of two separate resistive parts, or

$$R_{total} = R_r + R_l$$

The R_r is a resistance-like term called the radiation resistance which is a measure of how much wave energy is lost from the antenna by radiation per *rf cycle*.

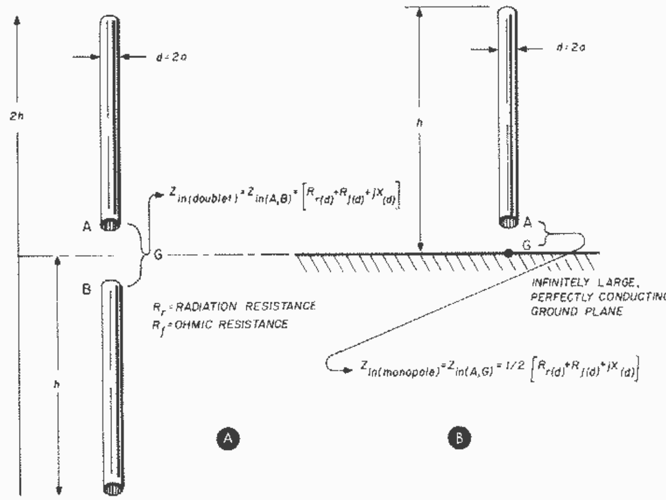

fig. 1. Center-fed, cylindrical doublet antenna in free space (A), and its feedpoint impedance. Equivalent monopole antenna of equal half-length, h, and equal radius, a, operated against an infinitely large, perfectly conducting ground plane is shown at (B).

No one wants the R_l ohmic loss resistance part of R_{total}. It just causes some of your input power to terminals A - B to be converted into heat, yet it is always present in real-world antenna elements. One of the battles in antenna design is to keep the ohmic part as small as possible in ways which will be discussed later. In any event, carefully log the input impedance,

$$Z_{in(A,B)} = R_{total(d)} + jX_{(d)}$$
$$= R_{r(d)} + R_{l(d)} + jX_{(d)} \text{ ohms}$$

which you have measured for this particular *doublet* antenna of specific half length h, and particular conductor radius a, at some exact rf frequency f (hertz).

Now imagine that the half of the antenna connected to input terminal B suddenly disappears, leaving only the other half-length element suspended in free space. A very thin, infinitely large, perfectly conducting metal sheet is then placed exactly through the mid-way point between the former input terminals, the sheet forming a plane lying at a right angle to the remaining antenna half (**fig. 1B**). The remaining antenna terminal A is now spaced a small distance above the metal plane. By connecting a ground lug to the metal plate at a point directly below terminal A, you now have a monopole antenna (half antenna) operating over a perfect ground plane. Label the newly installed ground terminal with the letter G. The remaining half of the former doublet is still as before: Same length h, same conductor radius a. With the rf bridge reconnected to the new input terminals A, G take a reading of the monopole input impedance over perfect ground. You find the new rf input impedance to be

$$\begin{aligned} Z_{in(m)A,G} &= \tfrac{1}{2}[Z_{in(A,B)}] \\ &= \tfrac{1}{2}[R_{total(d)} + jX_{(d)}] \\ &= \tfrac{1}{2}[R_{r(d)} + R_{l(d)} + jX_{(d)}] \ ohms \end{aligned}$$

or,

$$\begin{aligned} Z_{in(m)A,G} &= R_{total(m)} + jX_{(m)} \\ &= R_{r(m)} + R_{l(m)} + jX_{(m)} \ ohms \end{aligned}$$

As a result of this first experiment, you write yourself a rather formal note, "The complex input impedance $Z_{in(m)A,G} = R_{r(m)} + R_{l(m)} + jX_{(m)}$ of a cylindrical monopole antenna of conductor length h and conductor radius a, erected normal to an infinitely large, perfectly conducting ground plane, is exactly *one-half* the complex input impedance $Z_{in(A,B)} = R_{r(d)} + R_{l(d)} + jX_{(d)}$ of a full doublet antenna of identical half-length h and conductor radius a in free space, when both antennas are measured at the same radio frequency."

With this important experiment out of the way, step back some distance from the monopole antenna erected over the perfect ground plane so that you can inspect its entire length, h. Now, really using your imagination, assume that you own a very special pair of eye glasses which permit you to actually "see" electric field lines of force E, and magnetic field lines of force H. Carefully watching the monopole antenna, again turn on the rf generator so that it supplies energy at frequency f to the monopole input terminals A - G. **Fig. 2A** is an attempt to show what you would "see."

At the instant you closed the switch (t=1), a small expanding surface like a bubble would appear around the antenna base. Its surface would be covered with dotted E lines pointing radially outward from the surface of the monopole conductor element, with each E line gracefully arching over so that it pointed directly down at right angles to the flat surface of the ground plane. At the same instant you would preceive dashed circles of magnetic field lines of H form concentrically around the monopole antenna element, each growing larger in diameter with the passage of time. Let time suddenly freeze at this point so that you can closely inspect the initial wave surface.

The surface of the "bubble" is a wave front. As this is the first wave to be introduced to the monopole antenna, it's called a *precursor*; a sort of "scout wave" sent out to explore the electrical nature of the yet-unknown antenna to determine — at this one particular frequency — just exactly how the waves to follow will have to finally arrange themselves to be in agreement with certain natural laws.

One of these natural laws dictates that the electric lines of force always point precisely at right angles into the surface of a good conductor such as the antenna element, *and* also point precisely into the flat perfectly conducting surface of the groundplane. Another thing: The "antenna" is the total *combination* of the monopole conductor and the ground-plane surface; the wave front or antenna field is not *in* the antenna conductors, but instead fills the space surrounding the monopole conductor element and the ground-plane surface. The antenna conductor monopole element (or each half of the doublet in free space), together with the ground plane, compose the "nature" of the antenna, and are called the antenna *boundaries*. These boundaries are what the first precursor wave is trying to explore, for they *alone* will determine what finally happens later in time.

Now unfreeze time and let the wave front expand and climb higher up the antenna. Again freeze time at t=2. Now you will notice a very interesting effect: In order to span the increasing distance along the arc between the monopole conductor surface and the ground plane, the E lines get longer and longer as they climb up the monopole. You will also notice that the brightness of the E line arcs closest to the base of the antenna are less intense than those stretching over to ground from higher up on the antenna. Conversely, a fixed radial distance from the antenna, the magnetic field circles around the monopole conductor are intensely bright and glowing around the antenna base, but are less bright as they form around higher parts of the monopole conductor. Clearly, electric field intensity is *increasing* with height up the antenna; magnetic field intensity is *decreasing* with height.

Antenna specialists use the ratio of the magnitude of the electric line of force, E, to that of the magnetic field line, H, (at any point in space) to define what is called wave impedance, Z_W. This is comparable to what the electronics engineer merely calls impedance when he is dealing with the ratio of voltage, V, to current, I, in circuits physically small in terms of the wavelength of rf energy circulating within them. In contrast, antennas are "big" circuits in terms of the operating wavelength. As a consequence, their fields extend out to great distances around the antenna and the field — rather than voltage or current on conductors — is of first importance.

With this idea of wave impedance in mind, a re-examination of the monopole antenna field discloses that the wave impedance equal to E/H must be increasing in the wavefront as it expands higher and higher up the antenna (because E is increasing and H is decreasing). The wave impedance is small in magnitude near the input terminals A - G, but increases steadily with antenna height, h.

This brings up one important way of thinking about *all* antennas: Antennas attempt to perform an impedance-matching function, providing an impedance match between their input terminals and that of the surrounding space by using their conductors as wave impedance "transformers." In this picture, space itself is a common "master" transmission line connecting your antenna with every other antenna in the universe. Such a space transmission line has its own characteristic impedance, Z_s, and possesses an infinite number of input and output terminals. For the moment, however, just assume that this space transmission line surrounds your antenna; it wants to accept the rf energy you are putting into the input terminals of the antenna, but will only accept your wave energy as radiation when certain precise *boundary conditions* are met. The antenna tries to accomplish this feat of getting its waves off into space, but must wait to see what the precursor wave "says" after exploring the antenna.

Let time again unfreeze, and watch the wave front expand through t=3 to the moment of truth at t=4. The wave front has been racing (when you permitted it to) up the antenna at almost, but not quite, the speed of light (3 x 10^8 meters/second). Things appear to be going smoothly so far as the precursor wave is concerned, with those antenna boundaries changing in a nice, gentle fashion. Then *crash*! As if it had smashed head-on into a wall, the precursor wave finds the end of the monopole conductor. To the scout wave this is like an electromagnetic explosion. Let's freeze time again just at the instant this explosion occurs. What happened? To find out, we have to take a cut through the antenna field from a point looking directly down on the monopole conductor. Such a field cut is denoted by the dashed line x-y in view t=4 of **fig. 2**. The downward looking view is seen in **fig. 3**.

There are those outward pointing radial electric field lines, E, and the closed circles of magnetic lines, H. Waves in which the electric field lines lie at right angles to the magnetic field lines entirely in the plane of the wavefront, so that there are no electric or magnetic field components in the direction of wave propagation, are called *transverse electromagnetic* or TEM modes in wave shorthand. **Fig. 3B** shows the wave front inside an ordinary coaxial transmission line with air as an insulator. The similarity of the waves on the antenna and those in the non-radiating coaxial transmission line is no coincidence. Both are type TEM mode waves. In a TEM mode wave, the electric field lines *must end* on the surface of conductors; the monopole antenna conductor and the ground plane serve the same boundary purpose as the inner conductor and inside surface of the shield in the coaxial transmission line. In both cases, the wave fronts are guided in the empty space by the ending of the electric lines onto the oppositely polarized conducting surfaces. The TEM mode waves are held to these boundaries in the same manner as a spider with sticky feet when running around his web.

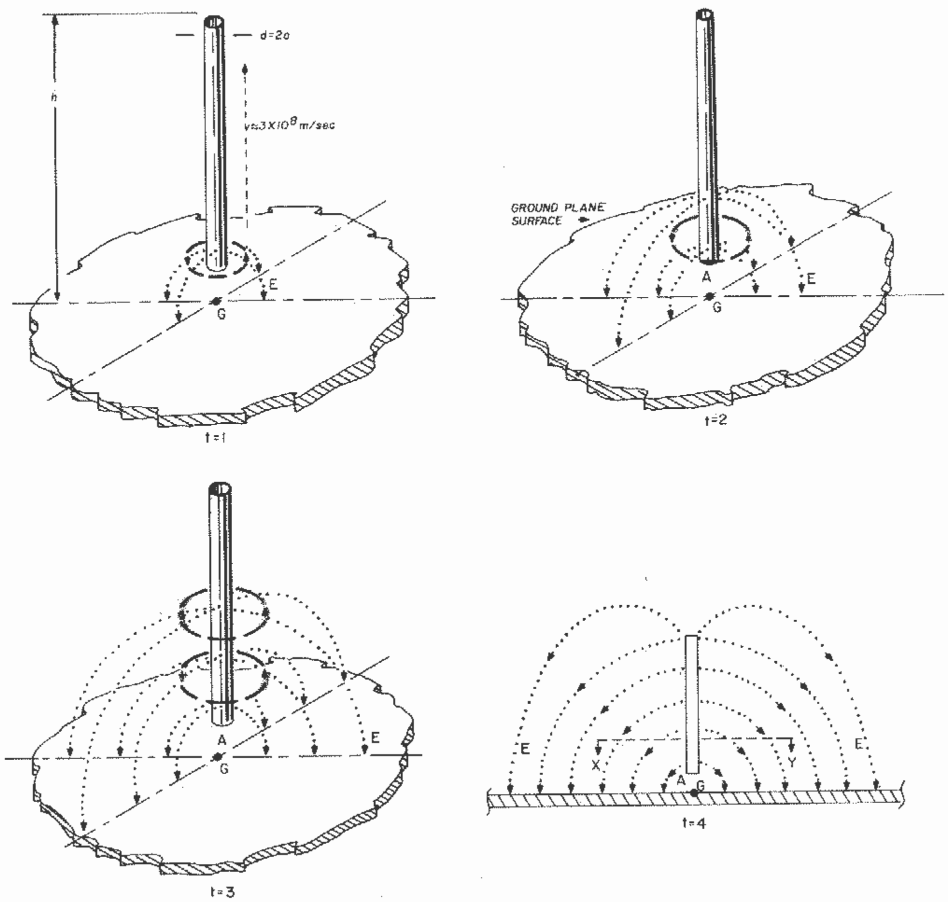

fig. 2. Precursor TEM wavefront moving up a cylindrical monopole antenna and outward on ground plane G for times t=1, t=2, t=3, and t=4. End reflection occurs at t=4. Top view of the E and H lines at t=4 are shown in fig. 3.

Now let's try to think the way an antenna theoretician does. Here we have these two classes of waves: TEM mode waves on the antenna in which all electric lines must end on conductors, and space or radiation waves. In free space (say halfway to the planet Mars) there are no electrical conducting surfaces upon which the electric field lines in space waves can end. But we already know that radio waves *can* propagate through free space. That reflected from the open end of the antenna; absolutely no wave energy got away into space as radiation. When a total reflection occurs, the only thing that prevents the space standing wave from reaching a vswr of infinity to one is the very small ohmic resistance of the highly conducting antenna element. Obviously, if that situation continued, antennas, as we know them would not exist. Fortunately, for radio amateurs and antenna men, it

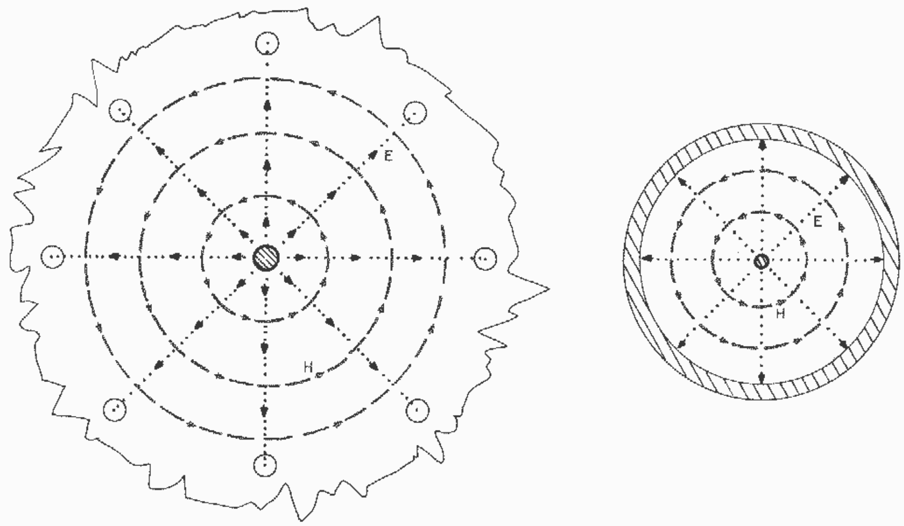

fig. 3. View looking down on the antenna field at x-y plane indicated in fig. 2. Note similarity to the E and H lines of the wavefront in a coaxial transmission line to right; both fields are type TEM.

must mean that the kind of waves which can exist as radiation must — regardless of wavefront geometry — contain electric lines which *close on themselves* or form loops the way the magnetic field closes on itself around our antenna. This idea turns out to be correct. There is an infinite variety of space wave modes, but *none* of them includes the TEM mode wave. Therefore, a pure TEM mode *cannot* make the transfer from the antenna boundaries into free space. Free space is an incompatible boundary condition for the TEM mode, precursor wave. Such incompatibility constitutes a huge *impedance mismatch* to the guided TEM mode wave at the *end* of the antenna.

Faced by a large mismatch at the top of the antenna, the TEM mode wave does what all waves do when faced by a mismatch on a transmission line: it is reflected and starts back down the antenna toward the input terminals. In doing so, however, the scout wave encounters other TEM mode waves coming up the antenna in the opposite direction. This kind of situation, with coherent waves moving in opposite directions, always produces the same phenomenon: Standing waves. Note, however, that these standing waves exist in space along the entire length of the antenna and are *not* to be confused with similar standing waves which can form in an antenna feedline because of an impedance mismatch between the antenna input impedance and the line's characteristic impedance.

It should be noted that the TEM scout wave is *totally* doesn't. Nature has arranged things so that as the downward moving scout wave continues to interfere with more and more TEM mode waves coming up the antenna, a wave conversion results; some of the original TEM mode energy is transformed into new, higher order mode waves — wave types which possess closed E and H line geometry and which can make the transfer from the antenna to space. As this converted wave energy (a surprisingly small amount of the total) begins to leave the antenna, energy loss causes the near-to-infinite vswr of the space standing wave to drop to a more reasonable magnitude. The antenna has now reached its steady state of operation.

Here, now, is our antenna: It is "ringing" like a sort of electromagnetic bell as the waves (not charges) race out along the length of the antenna, smash into the top end impedance discontinuity, then race back down the antenna, performing the mode change and supporting the existence of the space standing wave. Each rf cycle produces a small loss of energy to free space as radiation. The actual amount of radiation loss per rf cycle is dependent upon the length of the antenna (h or $2h$) in electrical degrees at the operating frequency, and the conductor geometry (which, for a monopole, includes the ground plane).

Do I hear you say that this picture sounds very much like one describing the way an open-ended (open-circuited) rf transmission line operates? Let's examine that idea! We saw that the wave impedance, $Z_W = E/H$, was not uniform along the length of the antenna. We also compared a cross section of the antenna field (precursor or scout wave) to the field inside a coaxial transmission line and found them to be the same. Now, even elementary books tell us that, in a lossless transmission line, the ratio of *distributed* series inductance to *distributed* shunt capacitance per unit length solely determines the characteristic impedance of the line as

$$Z_o = \sqrt{L/C}$$

(Advanced textbooks go on to say that the wave impedance, Z_W, of the TEM mode wavefront propagating down the transmission line is also a function of this same L/C ratio in the transmission line. Standard types of rf transmission lines, however, have uniform characteristic impedance so therefore they must have a *constant* L to C ratio per unit length.

Such reasoning makes it clear that the cylindrical antenna – when viewed as an rf transmission line – must possess a *variable* ratio of L to C along its length. To reinforce this idea, make another mental experiment: cut out a short section from the monopole antenna conductor. Measure the shunt capacitance to ground of this short conductor section, first at the antenna tip height, then at the antenna midheight, and finally, at the base just above the ground plane. Intuition tells us that the shunt capacitance to ground of the conductor section will be maximum at the antenna base, less at the midheight, and least at the top of the monopole. **Fig. 4** shows this same "measurement" result for the case of a doublet antenna. If capacitance to ground (or to the other side of a doublet) varies with position, obviously the L/C ratio cannot be a constant – and that says that the antenna characteristic impedance must also be non-uniform.

But, couldn't we just take this non-uniform characteristic impedance of the antenna and use it as a transmission line model of the antenna in the analogue key method? Yes, but this approach would be a bit messy to put into practical use. Calculations for non-uniform impedance transmission lines are more laborious than those related to lines with uniform impedance. Let's try again. If you have a quantity which changes in some smooth way over a given distance such as h, it's possible to take its mean or average value. In high class mathematics, this is called the integral of something (in this case, Z_o) over the length, h. That is actually what is done: You take this mean or average value of Z_o for the antenna length, h, and then use this *average* Z_o as the *uniform* Z_o of your analogue transmission line representing your antenna.

It sounds neat, except that getting this mean Z_o for an antenna is not an easy task. It was solved back in the 1920s at great calculation labor using a dc potential method. Fortunately, later work by Dr. Schelkunoff of Bell Telephone Laboratories has given us some simple formulas to determine the average characteristic impedance of certain kinds of antenna conductor geometry. These conductor geometries include those most often used by amateurs, and professionals alike.

Before presenting these simple formulas, let's make sure we have the concept of the analogue transmission line idea clearly in mind so it may be used with confidence in our experiments on paper with antennas.

antenna into transmission line

An rf transmission line constructed from extremely high conductivity elements of copper or aluminum, with only air as insulation, represents a very low electrical loss system. Electromagnetic waves moving down such a line stay almost perfectly constant in strength or amplitude even when traveling over long distances in electrical degrees of line length. This constancy of amplitude means that you can use simple trigonometric functions such as the sine, cosine, tangent, or cotangent of the line length in electrical degrees to accurately represent the behavior of waves on low loss line.

On the other hand, if you used poor conductors such as steel or lead to build an rf transmission line, the resistance of the conductors would rob energy from the waves moving down the line and convert it into heat; as a consequence of this energy loss, the wave amplitude would decay or decrease in strength with electrical distance traveled. To represent decaying waves you have to use mathematical functions which also decay in amplitude with electrical distance: Hyperbolic functions. Finally, in correctly representing radiation loss you come up against the cosine and sine integral calculus functions. Not only are these, valuable as they are, a little tacky to use in an amateur technical journal, but I promised at the beginning that only simple math would be needed.

A decision to stick to the use of simple, everyday trig functions means that we must use a lossless equivalent transmission line to represent the antenna. Knowing that real antennas have loss, hopefully the good kind of loss called radiation resistance, how can a lossless transmission line model of the antenna give us accurate answers when solving real antenna problems? Recall the TEM wave mode which did not radiate? That TEM mode would be a uniform amplitude wave representing the major portion of the rf energy oscillating (standing) in the antenna region. If we used only this non-radiating mode wave in the analogue line representing the antenna, the answers would describe *only* the reactive behavior of the antenna at its input terminals. The real or resistive part of Z_{in} would be missing in the answer because it is radiation energy loss which the TEM mode cannot account for in antenna systems. Is that bad? Certainly not! One of the most important things you want to find out when exploring your antenna ideas on paper is how the reactance at the input terminals will change as you move your transmitting frequency over an amateur band, what jX will do if you use a loading coil in the antenna, or how jX changes from one amateur band to another.

The real or resistive part of the antenna's input impedance (which is related to radiation resistance) changes very *slowly* with frequency; the reactive part, however, varies at a much greater rate with changes in operating frequency. *The rate at which the reactive part of antenna input impedance changes with frequency is* *governed by the antenna's characteristic impedance* when viewed as a transmission line. Does this mean that you just forget all about the real part of Z_{in}? No, not at all. You'll end up with a complete answer for $R_A + jX_A$ alright, but you will obtain the real part, R_A, the lazy man's way: By looking up the antenna's radiation resistance, R_r, as a function of its electrical length h (or $2h$) at the operating frequency, using published graphs of this data. Then you'll add the radiation resistance to the reactive part obtained from the analogue key transmission line key model. It's that simple if a) you are using high conductivity antenna conductors such as copper or aluminum, and b) are feeding the antenna at a current maximum point such as the base of a monopole or the center of a doublet. In the rare case where you are not feeding at a current maximum point on the antenna, then you transfer the R_r value you looked up to the actual feedpoint by a method to be given later.

Incidentally, in deference to Dr. Schelkunoff, it is only fair to mention that in the equivalent transmission line method he evolved, both the real and reactive parts of the total complex input impedance are obtained by using a special lumped "load impedance," determined by separate calculations, which is placed across the end or "output terminals" of the paper analogue transmission line representing the antenna. This more sophisticated

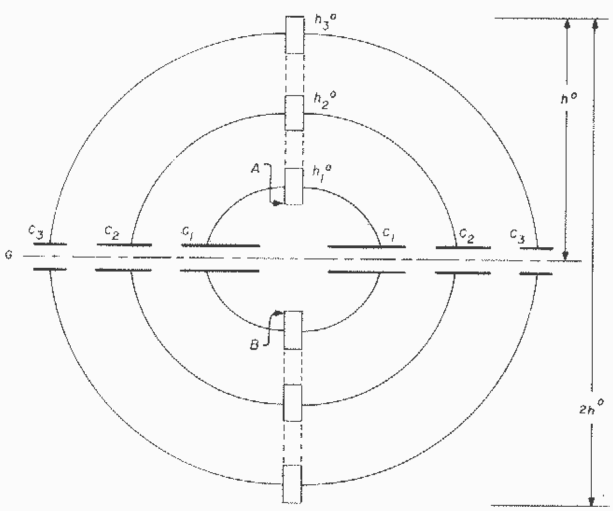

fig. 4. Variation in shunt capacitance between equal length conductor sections located at the input terminals, mid half-length, and tips of a doublet antenna. Passing the ground plane through G gives the same effect for an equivalent monopole. For a monopole, shunt capacitances double in magnitude.

technique, however, demands use of advanced forms of mathematics.[2]

characteristic impedance of cylindrical antennas

Schelkunoff gives the mean characteristic impedance of a doublet antenna with cylindrical conductor elements as,

$$K_A = 120\,(\log_e \frac{2h}{a} - 1)\ ohms \qquad (1)$$

Then, recalling the first experiment where you found that a monopole antenna of length h over a perfect ground plane had an input impedance exactly one-half that of a doublet antenna in free space of half length h, and the *same* conductor radius a, the mean characteristic impedance of a monopole antenna over ground is

$$K_m = 60\,(\log_e \frac{2h}{a} - 1)\ ohms \qquad (2)$$

Don't let the $\log_e$ part bother you. If your pocket electronic calculator doesn't give the natural $\log_e$ of a number directly, or if you only have tables of the common $\log_{10}$, then

$$\log_e \frac{2(h)}{a} = (2.3026) \times \log_{10} \frac{2(h)}{a}$$

The notation K_A and K_m is used to denote the mean antenna characteristic impedance instead of, say, Z_{oA} or Z_{om}. This avoids any confusion with the Z_o of a standard transmission line used to feed the antenna.

coming up

In the second part of this article I will describe use of the transmission line key method to solve a number of different antenna problems faced by the radio amateur. These will include the design of monopole and doublet antennas capable of being operated over the entire frequency width of an amateur band while keeping the vswr in the feedline down to a specified maximum value into which modern transmitters will load full power. I will also discuss base, center, and higher position coil loading of electrically short monopole and doublet antennas for maximum efficiency. Finally, I will show you how to "dissect" an antenna of your own design into parts to detemine if it will operate as you wish.

Each example will be carefully worked out in full detail (no steps omitted) so you can easily follow the solution and not get lost. In this way you will be able to quickly translate the analogue method to your own problems for any antenna on any amateur band. In the meantime, if you are totally unfamiliar or a bit rusty in the use of elementary plane vectors (phasors) to represent a complex ac impedance, $R + jX$, I suggest you visit the library and get a copy of *Basic Mathematics For Electronics*,[3] or its much earlier version, *Mathematics For Radiomen and Electricians*, by N. Cooke. Cooke was able to teach tens of thousands of Navy gobs to easily master basic ac math on a crash basis during WW II. You will also find him easy to follow and understand.

The difference between an amateur and a professional in a given field of science should not be one of knowledge, but only that the amateur is rewarded in pleasure and the professional in coin of the realm.

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